Lie Groupoids and Poisson Geometry

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Course Description: The classical Poisson bracket was first introduced by Poisson in the early nineteenth century in his study of the equation of motion in celestial mechanics. Poisson manifolds naturally appear as phase spaces of classical mechanical systems. In the last two decades, the theory of Poisson geometry has gone tremendous developments due to its close connection with many fields in mathematics including non-commutative differential geometry, Lie groupoid theory, integrable systems, quantization, quantum groups, and infinite dimensional Lie algebras and Gelfand-Fuchs theory. The purpose of this course is to give an introduction to the subject, assuming some basic knowledge of differentiable manifolds.

Topics:

1. **Poisson manifolds**: Lie-Poisson brackets, Poisson manifolds, Hamiltonian vector fields, Poisson cohomology, local structures of Poisson manifolds, symplectic foliation, Weinstein splitting theorem and the linearization problem, Coisotropic calculus

2. **Poisson group theory**: Lie bialgebras, Poisson groups, classical Yang-Baxter equation and r-matrices, dressing actions, Poisson group structure on semi-simple Lie groups, momentum map theory of Poisson group action, Manin triple, quasi-Lie bialgebras and quasi-Poisson groups

3. **Lie algebroids**: Lie algebroids and examples, Lie algebroid cohomology, Lie algebroid morphisms, Gerstenhaber and BV-algebras, Modular vector fields of Lie algebroids, Dirac submanifolds

4. **Lie groupoids**: Lie groupoids, bisections of Lie groupoids, Hilsum-Skandalis morphisms and Morita equivalence, the transformation groupoid associated with a group action, Lie algebroid of a Lie groupoid, exponential map, de Rham cohomology of Lie groupoids, groupoid cohomology and van Est isomorphisms

5. **Poisson groupoids**: Poisson groupoids and Lie bialgebroids, multiplicative multi-vector fields and universal lifting theorem, Courant algebroids and L_∞-algebras, quasi Lie bialgebroids, quasi-Poisson groupoids

6. **Symplectic groupoids** Symplectic realizations, symplectic groupoids, non-linear gauge theory and Poisson sigma model, quasi-symplectic groupoids

and group valued momentum map theory, Prequantization and contact groupoids

7. **Generalized complex geometry** Derived brackets and Courant algebroids, exact Courant algebroids and Severa classes, Dirac structures, pushforward and pullback, Clifford algebras and spinors, generalized complex structures, generalized complex submanifolds, local structures, Poisson-Nijenhuis structures, generalized Kahler structures, symmetry and reductions

Recommended textbooks:

1. S. Michea and P. Xu, Lie groupoids and Poisson geometry, book in writing
2. A. Cannas da Silva and A. Weinstein, Geometric models for noncommutative algebras, AMS, 1999.
3. J.-P. Dufour, and N. T. Zung, Poisson structures and their normal forms, Progress in Mathematics, 242. Birkhuser Verlag, Basel, 2005.
4. I. Vaisman, Lectures on the geometry of Poisson manifolds, Progress in Mathematics, 118. Birkhaeuser, 1994.
5. K. Mackenzie, General theory of Lie groupoids and Lie algebroids, London Mathematical Society Lecture Note Series, 213. Cambridge University Press, Cambridge, 2005.
6. I. Moerdijk, and J. Mrcun, Introduction to foliations and Lie groupoids, Cambridge Studies in Advanced Mathematics, 91, 2003.
7. P. Libermann, and C.-M. Marle, Symplectic geometry and analytical mechanics, Mathematics and its Applications, 35. D. Reidel Publishing Co., Dordrecht, 1987.