

OPERATOR ALGEBRAS, K -THEORY AND GROUPOIDS

JEAN-LOUIS TU

COURSE DESCRIPTION

This course's main theme is K -theory of operator algebras, with a focus on group and groupoid C^* -algebras. Assuming basic knowledge of functional analysis, we will start by reviewing classical properties of C^* -algebras and (topological) K -theory. Then, we will present two recent subjects of research: the Baum–Connes conjecture and twisted K -theory.

TOPICS

C^* -algebras. General theory. Representations. Group(oid) C^* -algebras. Crossed-products. Morita-equivalence. Nuclearity. Exactness. Amenability.

K -theory. General theory. KK -theory. Other cohomological functors: E -theory, cyclic (co-)homology.

The Baum–Connes conjecture. Statement. The Connes–Kasparov conjecture. Gromov's a -T-menability. Property T. Negative curvature and generalizations.

Twisted K -theory. General properties. Introduction to Freed-Hopkins-Teleman's theorem.

REFERENCES

- [1] J. Dixmier. Les C^* -algèbres et leurs représentations, Gauthier-Villars 1964.
- [2] G. Pedersen. C^* -algebras and their automorphisms, Academic Press 1979.
- [3] R. Kadison, J. Ringrose. Fundamentals of operator algebras, vol. I, II Academic Press 1983, 1986.
- [4] Renault, Jean. A groupoid approach to C^{**} -algebras. Lecture Notes in Mathematics, 793. Springer, Berlin, 1980.
- [5] M. Atiyah. K -theory, (W.A. Benjamin, New York, 1967).
- [6] B. Blackadar. K -theory for operator algebras, MSRI Publications, Vol. 5, (Cambridge University Press, Cambridge, 1998).
- [7] N.E. Wegge-Olsen. K -Theory and C^* - Algebras : A Friendly Approach. Oxford University Press (1993).
- [8] A. Connes. Noncommutative geometry. Academic Press (1994).
- [9] A. Valette. Introduction to the Baum–Connes conjecture. ETHZ lecture notes, Birkhäuser, 2001.
- [10] P. de la Harpe, A. Valette. La propriété (T) de Kazhdan pour les groupes localement compacts, Astérisque 175, SMF (1989).
- [11] M. Atiyah and G. Segal. Twisted K -theory, math.KT/0407054
- [12] D. Freed, M. Hopkins and C. Teleman. Loop Groups and Twisted K -Theory II. math.AT/0511232