

# Compact embedding of Sobolev spaces and well posedness of some optimization problems

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## Abstract

For every positive regular Borel measure, possibly infinite valued, vanishing on all sets of  $p$ -capacity zero, we characterize the compactness of the embedding  $W^{1,p}(\mathbf{R}^N) \cap L^p(\mathbf{R}^N, \mu) \hookrightarrow L^q(\mathbf{R}^N)$  in terms of the qualitative behavior of some characteristic PDE. This question is related to the well posedness of a class of geometric inequalities involving the torsional rigidity and the spectrum of the Dirichlet Laplacian, introduced by Polya and Szegö in 1951. In particular, we prove that finite torsional rigidity of an arbitrary domain (possibly with infinite measure), implies the compactness of the resolvent of the Laplacian and the compact injection of  $H_0^1(\Omega)$  not only in  $L^2(\Omega)$  but also in  $L^1(\Omega)$ .