Groupe de travail transfrontalier des doctorants

Cross-border workshop of Ph. D. students

December 3-4, 2008

Université Paul Verlaine-Metz

PROGRAM

Date and time	Speaker	Title
Dec 3rd, 13:30		Opening
Dec 3rd, 13:45	M. Geist	Kalman Temporal Differences
Dec 3rd, 15:00	C. Charignon	Buildings
Dec 3rd, 16:00		Coffee break
Dec 3rd, 16:45	F.-O. Schreyer	Groebner Basics and Applications
Dec 3rd, 19:30		Dinner
Dec 4th, 9:30	M. Ammar	Deformation Quantization
Dec 4th, 10:30		Coffee break
Dec 4th, 11:00	T. Pohlen	Universality: An Overview and Prospect
Dec 4th, 12:00		Lunch break
Dec 4th, 13:45	D. Ye	Prescribing the Jacobian Determinant

ABSTRACTS

Kalman Temporal Differences

Mathieu Geist

(Ph. D. student at Supélec and Arcelor)

Dec 3rd, 13:45

Optimal control of stochastic dynamic systems may be very complex. Even with a perfect knowledge of the system physics, optimal control policies may be impossible to determine analytically. Usual industrial responses to this issue are heuristic-based and rely on human knowledge. On another hand, the machine learning answer is known as Reinforcement Learning. The control problem is then described in terms of states, actions and rewards. In this framework, an artificial agent tries to learn an optimal control policy through real interactions with the system. It observes the state of the system and chooses an action to apply on it accordingly to a current internal policy mapping states to actions. A feedback signal is provided to the agent after each interaction as reward information, which is a local hint on the quality of the control. This reward is used by the agent to incrementally learn the optimal policy, simply by maximizing a function of the cumulative rewards. Commonly, the knowledge of the agent about the system is modelled as a so-called value function mapping states to an estimate of the expected cumulative reward. The optimal value function maps each state to its maximum expected cumulative rewards and the role of the agent can therefore be summarized as learning this function through interactions.

When state and action space have a finite number of elements, methods exist to compute the optimal value function, and thus the optimal policy. When these spaces are too large, an approximate representation of the value function should be chosen. However, reinforcement learning induces much more difficulties than classical function approximation. Notably the target (value) function is not directly observed, just the rewards are available. We mainly address the value function approximation problem for deterministic dynamic systems. A general statistical framework based on Kalman filtering paradigm is introduced. Its principle is to adopt a parametric representation of the value function, to model the associated parameter vector as a random variable and to minimize the mean-square error over parameters conditioned on past observed rewards. Combined with an approximation scheme called the unscented transform, the proposed Kalman Temporal Differences framework allows deriving a family of algorithms. They allow computing approximate value function, but also to handle non-stationarities and computing generalization uncertainty over the values, which can be useful for reinforcement learning (control and exploration/exploitation dilemma).

Buildings

Cyril Charignon

(Ph. D. student at the Elie Cartan Institute of Nancy , IECN)

Dec 3rd, 15:00

Geometry is often a useful way to study a group. As an example, the representation theory searches for all the way to have a group G acting on a vector space. But why have our group acting only on preexistant spaces? Building theory allows to directly create some topological spaces on which G acts, and wich reflect quite precisely the structure of G.

Groebner basics and applications

Frank-Olaf Schreyer

(Professor at the Saarbrücken University)

Dec 3rd, 16:45

The talk starts with the basic theory of Groebner basis and proceeds to somewhat more advanced applications, such as construction problems in mechanical engeneering.

Deformation quantization

Mourad Ammar

(Ph. D. student at the Paul Verlaine-Metz University and at the University of Luxembourg)

Dec 4th, 9:30

When switching from the Hamiltonian model of Classical Mechanics to the usual model of Quantum Mechanics, we completely change the nature of the observables. From functions on the phase space, we pass to operators on some Hilbert space. The commutator bracket $[-,-]$ of these operators substitutes for the classical Poisson bracket $\{-,-\}$ of functions.

The transition from Classical Mechanics to Quantum Mechanics is provided by Heisenberg's rules. These rules entail Dirac's equation $[\hat{f},\hat{g}] = ih\widehat{\{f,g\}}$, where f, g are functions of the phase space, $\hat{}$ denotes the quantization map, and h is Planck's constant. However, Van Hove's theorem (1952) states that this quantization cannot be extended to all phase space functions—and even not to all polynomials—, in such a way that Heisenberg's rules and Dirac's equation be still valid. The way out is to look for a quantization that verifies the weakened Dirac equation

$$[\hat{f},\hat{g}] = ih\widehat{\{f,g\}} + h\varepsilon(h), \tag{1}$$

where $\varepsilon(h)$ tends to 0 with h. Weyl's quantization W meets both requirements, Heisenberg's rules and Dirac's weak equation. Indeed, it quantizes any monomial in positions q_α and momenta p^α by the symmetrized product of the corresponding operators $\hat{q}_\alpha$ and $\hat{p}^\alpha$, which are of course given by Heisenberg's rules. The map W is not a homomorphism from classical to quantum observables, i.e. in general $W(f.g) \neq W(f) \circ W(g)$, where . is the pointwise product. One observes that $W(f) \circ W(g) = W(f \star g)$. Here $f \star g$ denotes the Moyal-Vey product

$$f \star g = f.g + \nu\{f,g\} + \sum_{k\geq 2} \nu^k c_k(f,g), \tag{2}$$

where $\nu = ih/2$ and where the c_k are bidifferential operators on the function space, say N, which vanish on constants and verify $c_k(f,g) = (-1)^k c_k(g,f)$. It is now easily seen that

$$[W(f), W(g)] = W(f \star g - g \star f) = ihW(\{f,g\}) - \frac{ih^3}{4}W(c_3(f,g)) + \ldots, \tag{3}$$

so that condition (1) is actually satisfied. Moreover, the Moyal $\star$ product is a formal deformation of the associative algebra $(N, .)$ and leads via antisymmetrization to a formal deformation of the Poisson algebra $(N, \{-,-\})$.

A seminal idea of Flato is that our description of Physics, should evolve, when facing a paradox, to a higher level by means of an appropriate deformation. In this perspective, Bayen, Flato, Fronsdal, Lichnerowicz, and Sternheimer suggest around 1975, in a founding article that appeared in "Annals of Physics", to abandon the representation of classical observables by linear operators, and to construct a model of Quantum Mechanics via deformation of the algebraic structure of the observable space N. Roughly speaking, the task is the construction, on any symplectic or Poisson manifold, of a $\star$-product similar to Moyal's product. This $\star$-product then allows endowing the space $N[[\nu]]$ of all formal series in ν with coefficients in N, with an associative noncommutative algebra structure, as well as with a Lie algebra structure; these algebras are formal deformations of $(N, .)$ and $(N, \{-,-\})$ respectively. Hence, in Deformation Quantization, Quantum Mechanics appears as a deformation of Classical Mechanics, as a deformation from commutativity to noncommutativity, such that the trace of noncommutativity on the classical level is the Poisson bracket.

In this talk, I will discuss the developments of deformation quantization over the last 30 years.

Universality: An Overview and Prospect

Timo Pohlen

(Ph. D. student at Trier University)

Dec 4th, 11:00

From classical approximation theorems one can show that the set of entire functions is dense in various function spaces, equipped with appropriate topologies. But it is possible to find even "smaller" sets that are also dense. These sets are typically generated by a single function—a so-called universal function. The (countable) process of generating such dense sets is usually realized by operators that are applied to the universal function. We will present some classical universalities.

Another question is if universality is preserved under the application of certain operations. In this talk we will answer this question for the case of the Hadamard product.

If φ is a universal function, it turns out that the universal behavior of the Hadamard product $\psi * \varphi$ depends on the (analytical) properties of ψ. We will exemplify some of these dependencies.

Prescribing the jacobian determinant

Dong Ye

(Professor at the Laboratoire de Mathématiques et Applications de Metz)

Dec 4th, 13:30

Let $\Omega \subset \mathbb{R}^N$ be a smooth bounded domain and $f : \Omega \to \mathbb{R}$ a positive function satisfying:

$$\int_\Omega f(x)dx = meas(\Omega). \qquad (H)$$

A classical problem is to find a function $u : \Omega \to \mathbb{R}^N$ satisfying $\det \nabla u = f$ in Ω and $u(x) = x$ on the boundary $\partial\Omega$. The equation is trivial in dimension one, but difficult in higher dimension because it is strongly nonlinear and under-determined. In 1941, J.C. Oxtoby and S.M. Ulam proved that the problem has always a weak solution if $f \geq C > 0$. J. Moser proved in 1965 that if $f \in C^\infty\left(\overline{\Omega}\right)$, then there exist solutions u which are C^∞ diffeomorphism.

So a natural question is the following: Let f verify (H) and be in a given class of functions (as $C^k\left(\overline{\Omega}\right)$, $C^{k,\alpha}\left(\overline{\Omega}\right)$ etc.), what we can expect as the best regularity for a solution u? For example, given any continuous and positive function f, can we find a C^1 diffeomorphism of $\overline{\Omega}$ such that $\det \nabla u = f$?